[2] Garceau, T. V., "Flight Data Entry," Mitre Tech. Rept. 4037, Series 217, July 1967, The Mitre Corp., Washington, D.C.

[3] Garceau, T. V., "Real-Time Quality Control of Radar Data," Mitre Tech. Rept. 4037, Series 223, July 1967, The Mitre Corp., Washington, D.C.

[4] Beeker, G. G., "Automatic Tracking," Mitre Tech. Rept. 4037, Series 218, Rev. 1, Feb. 1968, The Mitre Corp., Washington, D.C.

[5] Dodge, P. O., "Dynamic Radar Data Simulation," Mitre Tech. Rept. 4037, Series 224, Rev. 1, Feb. 1968, The Mitre Corp., Washington, D.C.

[6] Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems," *Journal Basic Engineering Transactions, American Society of Mechanical Engineers*, Vol. 82D, March 1960, pp. 35–45.

[7] Smith, G. L., Schmidt, S. F., and McGee, L. A., "Application of Statistical Filter Theory to the Optimal Estimation of Position and Velocity on Board a Circumlunar Vehicle," TR R-135, 1962, NASA.

[8] Battin, R. R., *Astronautical Guidance*, McGraw-Hill, New York, 1964, pp. 303–317.

[9] Bucy, R. S. and Joseph, P. D., *Filtering for Stochastic Processes with Applications to Guidance*, Interscience, New York, 1968, pp. 137–149.

[10] Bonin, L. J., "An Optimum Filter for Determining the Trajectory of Maneuverable Objects," *Proceedings of the ION National Space Meeting on Space Navigation—Present and Future*, The Institute of Navigation, April 1969, pp. 89–105.

[11] Lowenhar, H., "Collision Avoidance: How: When?" *Space/Aeronautics*, Vol. 53, No. 3, March 1970, pp. 20–28.

APRIL 1972 J. AIRCRAFT VOL. 9, NO. 4

Modeling the Effects of Pilot Performance on Weapon Delivery Accuracy

CORNELIUS T. LEONDES*
University of California, Los Angeles, Calif.
AND
ROBERT R. RANKINE JR.†
Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio

A model of the pilot-aircraft system which can relate the pilot tracking performance attainable with specific aircraft dynamics to the over-all accuracy of tactical weapon delivery is required in order to realistically determine essential flight control system dynamic performance characteristics. The approach taken is to derive a linear expression for projectile impact error in terms of errors in the task variables which are directly under the pilot's control. A statistical model of the propagation of these pilot-induced errors into impact error is then developed by considering each of the pilot inputs to be random variable. An analytical model of the human pilot is used to estimate the tracking error from the flight control system and aircraft dynamics and the turbulence environment. The model is further refined as a result of piloted simulation studies to include important cross-coupling effects. The resulting model represents a method for relating impact accuracy to design of the manual flight control system and provides a technique for comparative evaluation of display, computation, and control aids to the tactical pilot.

Introduction

UNTIL recently, improvement of tactical weapon delivery accuracy had concentrated upon reduction of weapon anomalies, reduction of component tolerances, development of more accurate measuring devices, and development of fire control systems such as predictive displays which can continuously indicate a correct aim point to the pilot. It often was assumed without question that the pilot would be able to accurately track the target with the aim point if the aircraft had met specified flying qualities, and no further attention was given to the pilot tracking error contribution to projectile impact accuracy. Such an assumption was expedient, however, because no adequate analytical method existed to define the manually attainable tracking error as a function of the aircraft dynamics and combine it with the other weapon delivery variables to predict over-all accuracy.

McRuer and Jex[1] generalized a control-display theory based on analytical models of the human pilot to achieve a unified model of the manually-controlled aircraft for specific flying tasks. Clement et al.,[2] succeeded in identifying the elements of the theory for instrument landing. Jackson[3] used piloted simulation to allocate weapon delivery deficiencies to the fire control or flight control systems, and showed the importance of tracking dynamics to firing opportunity in the air-to-air gunnery role.

In this article a unified model is defined for tactical air-to-ground weapon delivery. The analysis process on which the model is based combines analytical pilot models[4] and dynamic stability and control analysis methods[5] with traditional weapon-delivery error-analysis procedures,[6] thereby permitting simultaneous evaluation of both the control and guidance aspects of manual weapon delivery in terms of impact error. The approach also accounts for the cross-correlation of error sources due to pilot correction of one variable with intentional deviation in another, as well as other coupling effects.

Received April 5, 1971; revision received November 15, 1971. This article is based on research supervised by C. T. Leondes and supported in part under AFOSR Grant 699-67. The research was conducted by R. R. Rankine during an Air Force sponsored education program at UCLA.

Index categories: Aircraft Handling, Stability, and Control; Military Aircraft Missions.

* Professor of Engineering and Applied Science. Member AIAA.

† Major, USAF, and Assistant Professor of Electrical Engineering.

The model permits concurrent, mission-oriented treatment of fire control and flight control problems as integral elements of weapon delivery, instead of as isolated subsystems. Consequently, deficiencies may be treated in the logical order of their contribution to an over-all measure of system performance which can account for the interaction of the elements of the system. Most important, however, is that the effects of tracking dynamics, which can be altered by flight control system changes, are placed in their proper perspective for the contribution they can make to achieving better accuracy.

Air-to-Ground Weapon Delivery with a Fixed Sight

The depressed-reticle sight is the only display system used for target tracking during air-to-ground delivery of conventional weapons in many U.S. Air Force tactical fighter aircraft. A depressed-reticle sight is an optical display device which produces a visual reference for the pilot by projecting a reticle image onto an adjustable combining glass at the top center of the instrument panel. By viewing the real target through the combining glass, the pilot can simultaneously view the reflected reticle image. The reticle image generally consists of an aim dot (pipper) at the center of a segmented circle. The line of sight through the aim dot can be depressed below the zero reference line of the sight by rotating a calibrated knob on the side of the sight unit.

When a bomb is released from the aircraft, its initial velocity vector is the same as that of the aircraft; however, due to the acceleration of gravity and the deceleration of drag, the bomb impacts short of the point where its initial velocity vector intersects the Earth. Consequently, the aim dot of the optical sight, which the pilot attempts to position onto the target by maneuvering the aircraft, must be depressed some angular amount below the velocity vector of the aircraft in order to display the correct bomb impact point.

Prior to attacking the target the pilot depresses the reticle image a fixed amount calculated from the probable bomb trajectory. This trajectory is further dependent on the selection of a nominal weapon release (pickle) flight condition. Obviously, the aim dot indicates the correct bomb impact point only when the aircraft is at the correct, preselected weapon release flight condition. Thus, successful air-to-ground weapon delivery with a depressed-reticle sight requires that the pilot fly a fairly consistent weapon delivery profile which will bring the aircraft to a preplanned weapon release flight condition corresponding to the preset sight depression angle. In particular, he must dive the aircraft toward the target in such a way as to achieve the altitude, airspeed, and dive angle at pickle that were the basis for his sight depression setting, or else the weapon's initial velocity vector magnitude and direction will be incorrect. At the same instant, he must have the pipper on the target, or else the position of the velocity vector will not produce a trajectory that intersects the target.

A pilot can accomplish the foregoing multidimensional task only after a great deal of experience has allowed him to accumulate a mental catalog of roll-in flight conditions and imaginary canopy marks which suit his style as well as his chosen delivery conditions. Although each individual pilot employs a different assortment of flying styles and learned techniques, the end objective is always the same: to get as close to the chosen release condition as possible with the pipper on the target. Consequently, weapon delivery performance is often linearly analyzed by expanding the impact error equations in a Taylor series about the nominal release conditions and neglecting the higher-order terms.[6] The first step in such a perturbation analysis of the effects of pilot-induced errors on weapon delivery accuracy is, then, to express the impact error in terms of pilot-controlled variables.

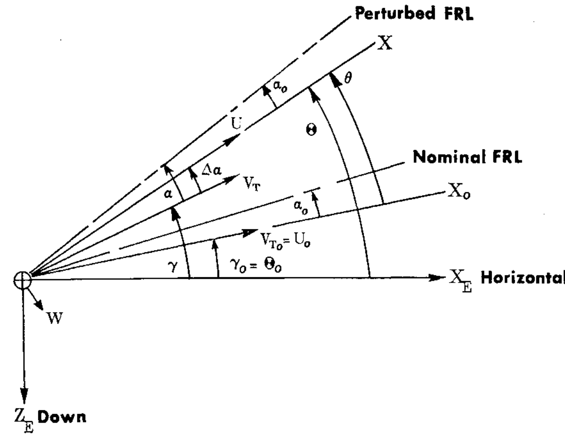

Fig. 1 Longitudinal perturbation of the aircraft velocity vector, $\mathbf{V}_T$, from an equilibrium position, $\mathbf{V}_{T_o}$. Stability axis notation, FRL = fuselage reference line.

Linear Models of Impact Error

Figure 1 indicates in stability-axis notation the effect of perturbing the aircraft velocity vector in the longitudinal plane from an equilibrium, weapon-release position. (All the angles are shown positive to eliminate confusion with sign; actually, the nose would be below the horizon making γ_o and Θ_o negative.) The equilibrium weapon release position is defined with the pipper on target, and, since the line of sight (LOS) through the aim dot is a fixed angle Σ below the fuselage reference line (FRL), a perturbation of the FRL is equivalent to a perturbation of the LOS. Thus, the longitudinal error in pipper placement perceived by the pilot is the perturbation pitch attitude of the aircraft, θ.

The longitudinal ballistic trajectory of the projectile is dependent on the velocity vector of the aircraft at the instant of pickle and on any other additional velocity or acceleration imparted upon the projectile itself. Since pilot-induced errors are of primary interest here, the problem will be simplified by assuming the average wind velocity to be zero and neglecting the drag deceleration of the projectile. These assumptions still give reasonably good results for bullets and low drag bombs. The initial velocity vector of the projectile is, therefore:

$$\mathbf{V}_i = \mathbf{V}_T + \mathbf{V}_m \tag{1}$$

where $\mathbf{V}_T$ = aircraft velocity vector at pickle, and $\mathbf{V}_m$ = the muzzle velocity of the ammunition (= 0 for bombs). By defining ε as the angle of gun depression below the fuselage reference line, Eq. (1) may be written in Earth coordinates using the notation of Fig. 1

$$\mathbf{V}_i = V_{x_i}\mathbf{i} + V_{z_i}\mathbf{k} \tag{2}$$

where

$$V_{x_i} = V_T\cos(\Theta_o + \theta - \Delta\alpha) + V_m\cos(\Theta_o + \theta + \alpha_o - \varepsilon)$$
$$V_{z_i} = -V_T\sin(\Theta_o + \theta - \Delta\alpha) - V_m\sin(\Theta_o + \theta + \alpha_o - \varepsilon)$$

and $\mathbf{i}$ and $\mathbf{k}$ are unit vectors along the X_E and Z_E axes, respectively.

The projectile will impact the Earth at a horizontal range r from the aircraft weapon release position after dropping a distance h where h is the weapon release altitude above ground level. Using elementary kinetics, the time of flight of the projectile can be written

$$T_F = [-V_{z_i} + (V_{z_i}{}^2 + 2gh)^{1/2}]/g \tag{3}$$

where g is the acceleration of gravity.

Therefore, the horizontal range of the impact point from the aircraft weapon release position is

$$r = V_{x_i} T_F \tag{4}$$

The range r_t of the target from the aircraft weapon release position is defined by the altitude and the LOS through the aim dot with pipper on target

$$r_t = -h/\tan(\Theta_o + \alpha_o - \Sigma) \tag{5}$$

Consequently, the longitudinal impact error, or range error, is

$$e_r = r - r_t \tag{6}$$

The correct sight depression setting for a particular weapon delivery profile can be determined by substituting the selected nominal weapon release values of h, Θ_o, V_T, and α_o into Eq. (6) and solving for Σ with e_r, θ, and $\Delta\alpha$ set equal to zero.

Equation (6) is a nonlinear function in the variables h, $\Theta_o V_T$, α_o, θ, and $\Delta\alpha$ which may be approximated by the linear terms of a Taylor series expansion about the nominal weapon release flight condition. First note, however, that trim angle of attack can be approximated as a linear function of altitude and airspeed in the vicinity of the nominal trim angle of attack; that is,

$$\alpha_o = \alpha_{on} + K_o(h - h_n) + K_1(V_T - V_{T_n}) \tag{7}$$

where the subscript n denotes the nominal, preplanned weapon release value of the variable. When error in trim angle of attack is eliminated from Eq. (6) by expressing it in terms of altitude and airspeed errors with Eq. (7), a linear approximation of range error of the following form results:

$$e_r \doteq K_u \Delta U_o + K_h \Delta h + K_\Theta \Delta\Theta_o + K_\theta \theta + K_\alpha \Delta\alpha \tag{8}$$

where $\Delta U_o = U_o - U_{o_n} \doteq V_T - V_{T_n}$ = airspeed error at pickle,

$\Delta h = h - h_n$ = altitude error at pickle

$\Delta\Theta_o = \Theta_o - \Theta_{o_n}$ = dive angle error at pickle

θ = tracking error at pickle

$\Delta\alpha$ = angle-of-attack perturbation from trim at pickle

$K_u = \left.\dfrac{\partial e_r}{\partial U_o}\right|_n$ = range error sensitivity to airspeed error at pickle

$K_h = \left.\dfrac{\partial e_r}{\partial h}\right|_n$ = range error sensitivity to altitude error at pickle

$K_\Theta = \left.\dfrac{\partial e_r}{\partial \Theta_o}\right|_n$ = range error sensitivity to dive angle at pickle

$K_\theta = \left.\dfrac{\partial e_r}{\partial \theta}\right|_n$ = range error sensitivity to longitudinal tracking error at pickle

$K_\alpha = \left.\dfrac{\partial e_r}{\partial \Delta\alpha}\right|_n$ = range error sensitivity to angle-of-attack perturbations from trim at pickle

Expressions for these sensitivities which result from taking the partial derivities of Eq. (6) are listed in Table 1 along with representative values for a strafing pass of a typical fighter aircraft. The values are the same whether computed in stability or body axes, although the expressions would differ.

The release altitude error (Δh) is essentially due to a pickle timing error of the pilot times the aircraft sink rate. Since the sink rate, airspeed, and dive angle are varying slowly in comparison with tracking error and angle-of-attack perturbations at the instant of weapon release, the first three terms of Eq. (8) may be considered error contributions due to (relatively) static differences between the actual "equilibrium" flight condition at pickle and the nominal, preselected weapon release flight condition. The last two terms of Eq. (8) then are error contributions due to dynamic perturbation from the actual equilibrium flight condition at pickle.

Analysis of lateral perturbation of the aircraft velocity vector proceeds similarly to the foregoing longitudinal analysis, and determines the effects of sideslip (β), yaw (ψ), and roll (ϕ) perturbations on lateral impact error or deflection. Such an analysis[7] results in an expression for the lateral tracking error (μ) observed by the pilot:

$$\mu = \psi - C \tag{9}$$

where $C = \sin^{-1}[\sin\phi \sin(\Sigma - \alpha_{o_n})]$, and an expression for the lateral (cross-track) deflection of the impact point from the target:

$$d = [U_{o_n}\sin(\psi + \beta) + V_m \sin(\psi - A)] \cdot T_{F_n} \tag{10}$$

where $A = \sin^{-1}[\sin\phi \sin(\varepsilon - \alpha_{o_n})]$. Equation (9) may be linearly approximated for small angles by the expression:

$$\mu = \psi - (\Sigma - \alpha_{o_n})\phi \tag{11}$$

When Eq. (10) is approximated by the linear terms of a Taylor series expansion about the nominal, preplanned weapon release flight condition, an expression of the following form results:

$$d \doteq K_\psi \psi + K_\beta \beta + K_\phi \phi \tag{12}$$

Table 1 Range error sensitivity parameters

Sensitivity	Expression in stability axes[a]	Typical value for strafing pass[b]
K_u	$[\cos\Theta_{o_n} - V_m K_1 \sin(\Theta_{o_n} + \alpha_{o_n} - \varepsilon)]T_{F_n} + (V_{x_{in}}/g)[\sin\Theta_{o_n} + V_m K_1 \cos(\Theta_{o_n} + \alpha_{o_n} - \varepsilon)]\{1 - (V_{z_{in}}/[V_{z_{in}}^2 + 2gh_n]^{1/2})\} - K_1 h_n \csc^2(\Theta_{o_n} + \alpha_{o_n} - \Sigma)$	.498 sec
K_h	$\{V_{x_{in}}/[V_{z_{in}}^2 + 2gh_n]^{1/2}\} + \cot(\Theta_{o_n} + \alpha_{o_n} - \Sigma)$	−.103
K_θ	$V_{z_{in}} T_{F_n} + \dfrac{V_{x_{in}}^2}{g}\{1 - (V_{z_{in}}/[V_{z_{in}}^2 + 2gh_n]^{1/2})\}$	269. ft/deg
K_Θ	$K_\theta - h_n \csc^2(\Theta_{o_n} + \alpha_{o_n} - \Sigma)$	−5.16 ft/deg
K_α	$U_{o_n} T_{F_n} \sin\Theta_{o_n} - \dfrac{V_{x_{in}} U_{o_n} \cos\Theta_{o_n}}{g}[1 - (V_{z_{in}}/[V_{z_{in}}^2 + 2gh_n]^{1/2})]$	−65.3 ft/deg

[a] The additional simplification that $K_o \doteq 0$ has been made here.
[b] Nominal values: $h_n = 500$ ft, $U_{o_n} = 520$ KTAS, $\Theta_{o_n} = -10°$ in stability axes, $\alpha_{o_n} = 1.9°$, $\varepsilon = 2°$, $K_1 = -1.1476(10^{-4})$ rad/ft/sec, $V_m = 2739.6$ ft/sec, which give $\Sigma = 2.172°$.

Table 2 Deflection sensitivity parameters

Sensitivity	Expression in stability axes[a]	Typical value for strafing pass[b]
K_ψ	$(U_{o_n} + V_m)T_{F_n}$	48.9 ft/deg
K_β	$U_{o_n}T_{F_n}$	11.9 ft/deg
K_ϕ	$-V_m T_{F_n} \sin(\varepsilon - \alpha_{o_n})$	$-.0646$ ft/deg

[a] K_ψ and K_β are the same in body axes; K_ϕ becomes $-V_m T_{F_n} \sin\varepsilon = -1.293$ ft/deg.
[b] See Table 1 for nominal condition description.

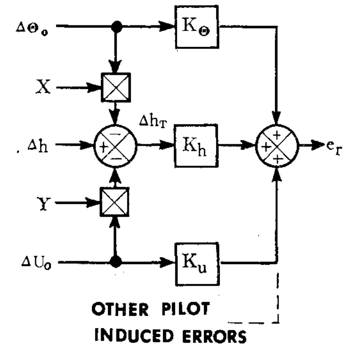

Fig. 2 Model of pilot dive angle and airspeed error compensation with altitude.

where ψ = dynamic perturbation of yaw angle at pickle,
β = dynamic perturbation of sideslip angle at pickle
ϕ = dynamic perturbation of roll angle at pickle
$K_\psi = (\partial d/\partial\psi)|_n$ = deflection sensitivity to yaw at pickle
$K_\beta = (\partial d/\partial\beta)|_n$ = deflection sensitivity to sideslip at pickle
and $K_\phi = (\partial d/\partial\phi)_n$ = deflection sensitivity to roll at pickle

Expressions for these sensitivities which result from taking the partial derivatives of Eq. (10) are listed in Table 2 along with representative values for the strafing pass described in Table 1.

Impact Error Variances

In order to estimate the range-error and deflection variances it is first assumed that each of the variables in Eqs. (8) and (12) is a random variable. Once a statistical estimate of the variance (σ^2) of each of these random variables is obtained, the variance of the range error and deflection could be easily calculated if the random variables were uncorrelated. If this were the case, the variance of the sum in Eqs. (8) and (12) would merely be the sum of the variances. However, this is not the case; the random variables in Eqs. (8) and (12) are correlated. The random variable $\Delta\alpha$ is related to the random variable θ by the longitudinal dynamics of the aircraft. The random variable Δh is related to the random variables ΔU_o and $\Delta\Theta_o$ because the pilot may intentionally introduce error in release altitude at pickle to compensate for observed errors in airspeed and dive angle. The random variables ψ, β, and ϕ are all related by the lateral-directional dynamics of the aircraft. Therefore, correct equations for the range-error and deflection variances must either include cross-correlation terms or else be expressed in terms of more basic, uncorrelated random variables. The latter approach will be used here because it gives insight to the origin of the errors.

It was explained earlier with respect to Fig. 1 that as the aircraft approaches weapon release, the pilot observes pitch attitude perturbation (θ) as the longitudinal difference between pipper position and the target. Theoretically, he would be able to observe total body-axis pitch attitude ($\Theta_B = \Theta_{o_B} + \theta$) from the artificial horizon of the attitude director indicator (ADI). However, the coarse resolution and low bandwidth of the displayed attitude signal diminish the information observable to merely equilibrium dive angle (Θ_{o_B}). Since the pilot knows the desired dive angle, the ADI may be considered a display of the equilibrium dive angle error ($\Delta\Theta_o = \Theta_o - \Theta_{o_n}$) which is the same whether measured in body or stability axes. Hence, $\Delta\Theta_o$ at pickle is a static random variable dependent on the pilot error in establishing an aircraft trajectory at roll-in which is consistent with his style and chosen nominal dive angle, and it is observed from the ADI. On the other hand, θ at pickle is a dynamic random variable dependent upon the pilot's tracking performance, and it is observed from the optical sight. Because of this independence in origin and observation, it will be assumed that $\Delta\Theta_o$ and θ are uncorrelated at the instant of weapon release.

The pilot often introduces intentional error in weapon release altitude to compensate for observed errors in dive angle and airspeed.[6] By his mental processing of the observed altitude, dive angle, and airspeed data, the pilot introduces a cross-feed of dive angle error and airspeed error into his altitude control. Thus, the total altitude error on a weapon delivery pass will be due to the combined effect of the intentional errors of dive angle and airspeed compensation plus the unintentional error of the pilot's ability to control the release altitude exactly. This process is to be modeled as indicated in Fig. 2 where X is a random variable representing the crossfeed gain of perceived dive-angle error into altitude error. For an experienced pilot the mean of X will be the correct crossfeed gain which is the ratio K_Θ/K_h obtained from Table 1. The variance of X, σ_x^2, is an indication of the imperfection of the pilot's mental calculation of the correction required. Similarly, Y is a random variable representing the crossfeed gain of perceived airspeed error into altitude error. For an experienced pilot the mean of Y will be the correct crossfeed gain which is the ratio K_u/K_h obtained from Table 1. The variance of Y, σ_y^2, is an indication of the imperfection of the pilot's mental calculation of the correction required. Δh_T is the altitude error that would be measured on any one pass, whereas Δh is that portion of Δh_T which is due only to the pilot's inability to pickle at precisely the altitude he mentally calculates. These fundamental input random variables are considered to be independent, and their contribution to the range error variance may be computed using methods for treating functions of random variables[8] to obtain

$$\sigma_{e_{r_s}} = \sigma_{U_o}{}^2 K_h{}^2 \sigma_y{}^2 + \sigma_{\Theta_o}{}^2 K_h{}^2 \sigma_x{}^2 + K_h{}^2 \sigma_h{}^2 \tag{13}$$

where $\sigma_{e_{r_s}}{}^2$ = the variance of the static portion of the range error due to pilot control of airspeed, dive angle, and altitude at pickle.

The relationship between θ and $\Delta\alpha$ and the relationship between ψ, β, and ϕ are expressed by differential equations and are assumed to be independent from each other and from the variables of Eq. (13). Perturbations of the longitudinal variables θ and $\Delta\alpha$ and the lateral-directional variables ψ, β, and ϕ are assumed to be the result of pilot target tracking in clear air turbulence. Complete analysis of the correlation and variance of these variables requires the introduction of gust, aircraft, and pilot analytical models. This analysis procedure is described briefly in the following paragraphs. A more detailed description is included in Ref. 7.

For the analysis of longitudinal tracking the dynamics of the plant that the pilot controls are determined from the short-period approximate aircraft equations and the equations of the elevator control system.[5] A lengthwise distribution of vertical gust velocities is assumed to aerodynamically disturb the angular and angular-rate components of the plant equations[9] to produce a continuous random pitch attitude perturbation. This perturbation, which may be represented as low-pass-filtered white noise, is the longitudinal tracking error input to the pilot. From these input and plant characteristics, an analytical pilot model is determined[4] which can include the effects of display scanning.[10,11] The pilot model is then combined with the plant dynamics to predict the mean-square intensity ($\sigma_{e_{rd}}^2$) of the dynamic portion of the range error where

$$e_{rd} = K_\theta \theta + K_\alpha \Delta\alpha \tag{14}$$

That is, the complete, pilot-in-the-loop transfer functions which relate θ and $\Delta\alpha$ to the white noise source producing the turbulence input are determined and combined according to Eq. (14). The power spectrum density of e_{rd} can then be calculated and integrated to obtain the mean-square intensity $\sigma_{e_{rd}}^2$.

θ and $\Delta\alpha$ are correlated random processes that the pilot samples at the instant of pickle. By the definition of a random process these particular values of θ and $\Delta\alpha$, sampled at the time of weapon release, are random variables. If the random processes θ and $\Delta\alpha$ are assumed to be ergodic so that time averages equal expected values, then the variance of the random variable e_{rd} will be equal to the calculated value of mean-square intensity, $\sigma_{e_{rd}}^2$.

For a better insight to the relative effects of tracking error on miss distance, an "effective" tracking error sensitivity can be calculated by dividing $\sigma_{e_{rd}}$ by the rms longitudinal tracking error, σ_θ; that is, let

$$K_2 = \sigma_{e_{rd}}/\sigma_\theta \tag{15}$$

= effective range error sensitivity to longitudinal tracking error at pickle (including effects of correlated angle-of-attack perturbations).

Since the random variables θ and $\Delta\alpha$ are presumably independent of the other random variables in Eq. (8), the complete variance of the range error may be obtained by adding the variance $\sigma_{e_{rd}}^2 = (K_2\sigma_\theta)^2$ to Eq. (13); that is,

$$\sigma_{e_r}^2 = (K_h\sigma_y\sigma_{U_0})^2 + (K_h\sigma_h)^2 + (K_h\sigma_x\sigma_{\Theta_0})^2 + (K_2\sigma_\theta)^2 \tag{16}$$

It should be noted from Eq. (16) that pilot correction of dive angle and airspeed errors with intentional release altitude errors has made the range-error variance independent of the previously-calculated sensitivities K_u and K_Θ. Instead, the range-error variance is dependent on the pilot's ability to use weapon release altitude to correct his observed dive angle and airspeed mistakes. Also, the fact that an observed longitudinal tracking error θ implies a particular $\Delta\alpha$ via the aircraft equations of motion results in a single variance term representing the combined effects of angle-of-attack perturbations and tracking error. Equation (16) expresses range-error variance in terms of the variances of the four longitudinal variables that the pilot observes and controls directly (airspeed, altitude, dive angle, and tracking error) and the two mental corrections of release altitude that he allegedly makes (airspeed and dive angle). The expression arises from the statistical properties of functions of independent random variables.

The lateral-directional variable that the pilot observes and controls is the lateral tracking error μ defined by Eq. (11). Control of μ by rudder pedal and lateral control stick motions results in implied control of yaw (ψ), sideslip (β), and roll (ϕ) through the aircraft equations of motion. When $\Sigma - \alpha_{o_n}$ is very small, as is the case for the example gun-fire pass introduced in Table 1, μ is approximately equal to ψ, and ψ may be taken as the lateral tracking error.

For the analysis of lateral tracking, the dynamics of the plant that the pilot controls are determined from the lateral-directional aircraft equations and the equations of the rudder and aileron/spoiler control systems. A spanwise distribution of vertical gust velocities is assumed to aerodynamically disturb the roll-rate components of the plant equations to produce a continuous random yaw attitude perturbation.‡ This perturbation, which may be represented as low-pass-filtered white noise, is the lateral tracking error input to the pilot. From these plant and input characteristics an analytical pilot model is determined. This pilot model is combined with the plant dynamics to predict the mean-square intensity (σ_d^2) of the lateral deflection of the impact point from the target. That is, the complete, pilot-in-the-loop transfer functions which relate ψ, β, and ϕ to the white noise source producing the turbulence input are determined and combined according to Eq. (12). The power spectrum density of d can then be calculated and integrated to obtain the mean-square intensity σ_d^2. Again, assuming the ergodicity of the random processes ψ, β, and ϕ, the variance of the random variable d will be equal to the calculated value of mean-square intensity, σ_d^2.

An "effective" lateral tracking error sensitivity can be computed analogously to the longitudinal case by dividing σ_d by the rms lateral tracking error, σ_ψ; that is, let

$$K_3 = \sigma_d/\sigma_\psi \tag{17}$$

= effective deflection sensitivity to lateral tracking error at pickle (including effects of correlated sideslip and roll angle perturbations).

so that the deflection variance may alternately be expressed:

$$\sigma_d^2 = (K_3\sigma_\psi)^2 \tag{18}$$

Model Refinements

Several piloted computer simulations of gun firing passes were conducted to confirm and refine the statistical model of propagation of pilot errors into impact errors represented by Eqs. (16) and (18). The simulation results, which are reported in Ref. 7, disclosed that for the strafing profile tested the pilots made no altitude corrections for dive angle and airspeed errors (i.e., $X = Y = 0$). In addition, significant cross-coupling was found to exist between lateral tracking, longitudinal tracking, and release altitude.

Dive Angle and Airspeed Corrections

The fact that the pilot does not bother to introduce release altitude corrections for observed dive angle and airspeed errors during strafing is easily understood by referring to Tables 1 and 2. The gun-firing error sensitivities, which the pilot learns implicitly through experience or explicitly through analysis, reflect the relative insignificance of dive angle, airspeed, and altitude errors in comparison with tracking errors and the variables dynamically coupled thereto. However, for dive bombing the effects of dive angle and airspeed errors are more significant making the reported pilot correction of such errors with release altitude deviation more effective and therefore more likely. Consequently, the mathematical representation of this pilot correction procedure depicted in Fig. 2 may prove practical for modeling the dive bombing mission, but it is not required for strafing analysis. That is, dive angle, airspeed, and altitude errors may be presumed independent for air-to-ground gun-firing analysis.

Homogeneity

Since a pilot tends to perform homogeneously in each axis of a two-axis tracking task,[12] a better representation of pilot tracking performance will be obtained by determining a single pilot model expression which is analytically feasible for both axes. Since the lateral tracking task must often be accomplished with a lower bandwidth than the longitudinal tracking task due to stability constraints, the lower gain lateral tracking pilot model is used in both axes. (This is in agreement with pilot comments of "loose" tracking in both axes during air-to-ground weapon delivery, even though fairly "tight" control of longitudinal tracking appears possible from single-axis analysis.)

‡ Side gusts are often used for lateral tracking analysis, but the spanwise variation of the vertical gust distribution can also produce a significant lateral response of the airplane.[9] For further clarification of this point see the article "Airplane Yaw Perturbations due to Vertical and Side Gusts" beginning on page 316 of this issue.

Visual-Motor Interference

The lateral tracking dynamics of the tactical fighter aircraft used for the piloted computer simulations of strafing were considerably less favorable than the longitudinal tracking dynamics. The larger lateral tracking errors which resulted interfered with the pilot's ability to sense error and error-rate in the longitudinal axis; consequently, longitudinal tracking performance was degraded from that predicted through single axis analysis, even with homogeneous pilot models. In addition, cross-coupling between the lateral and longitudinal axis may have been introduced manually when control movements intended for one axis of the two-axis control stick inadvertently produced motion in the other. Levison and Elkind[12] term these cross-coupling effects "visual-motor interference" and suggest that when the disturbance inputs to the two axes are uncorrelated and the pilot's transfer functions are nearly the same in both axes, the coupling is proportional to the mean square tracking error. Therefore, degradation of the longitudinal tracking performance by the poorer lateral tracking performance may be represented:

$$\sigma_{\theta_{2-\mathrm{axis}}}{}^2 = \sigma_{\theta_{1-\mathrm{axis}}}{}^2 + K_{\theta\psi}{}^2\sigma_\psi{}^2 \tag{19}$$

where $K_{\theta\psi}$ = interference coefficient.

The sensitivity of the range error to the longitudinal tracking error described by Eq. (19) is no longer simply K_2. Only the first term on the right-hand side of Eq. (19) is due to the lengthwise vertical gust distribution that causes angle-of-attack perturbations. The second term is not correlated with the angle-of-attack disturbances; consequently, it is related to the range error by the sensitivity parameter K_θ, not by K_2 which was derived to account for correlated angle-of-attack effects.

Firing Instant Variability

The piloted computer simulations of gun-fire passes also revealed that some of the pilot's variability in release altitude control could be attributed to firing a little before or after the desired release altitude was reached if the pipper was in good position. Again this relates to the fact that the impact error is more sensitive to tracking errors than it is to altitude errors. Since lateral tracking error predominated for the tactical fighter aircraft simulated, and total longitudinal tracking error is dependent on lateral tracking error as in Eq. (19), the altitude can be considered to be (approximately) coupled to lateral tracking error only. That is, altitude control is degraded by lateral tracking error:

$$\Delta h_T = \Delta h + K_{h\psi}\psi \tag{20}$$

where $K_{h\psi}$ is the lateral-tracking-error to altitude coupling coefficient. But this altitude degradation reduces the total effective lateral tracking error:

$$\psi_T = (1 - K_{h\psi}K_{\psi h})\psi \tag{21}$$

where $K_{\psi h}$ is the altitude to lateral-tracking-error coupling coefficient and $0 < (1 - K_{h\psi}K_{\psi h}) < 1$.

The coupling effects discussed in this section are summarized in Fig. 3. By assuming that Fig. 3 represents another function of several independent input random variables the following relations for the statistics of the range error and deflection can be derived

$$\sigma_{e_r}{}^2 = K_u{}^2\sigma_{U_o}{}^2 + K_\Theta{}^2\sigma_{\Theta_o}{}^2 + K_h{}^2\sigma_h{}^2 + K_2{}^2\sigma_\theta{}^2 + K_4{}^2\sigma_\psi{}^2 \tag{22}$$

where $K_4 = K_hK_{h\psi} + K_\theta K_{\theta\psi}(1 - K_{h\psi}K_{\psi h})$, and

$$\sigma_d{}^2 = (1 - K_{h\psi}K_{\psi h})^2K_3{}^2\sigma_\psi{}^2 \tag{23}$$

Similarly, the variance of the measurable altitude error at pickle will become

$$\sigma_{h_T}{}^2 = \sigma_h{}^2 + K_{h\psi}{}^2\sigma_\psi{}^2 \tag{24}$$

In these equations $\sigma_\theta{}^2$ and $\sigma_\psi{}^2$ are the analytically predicted,

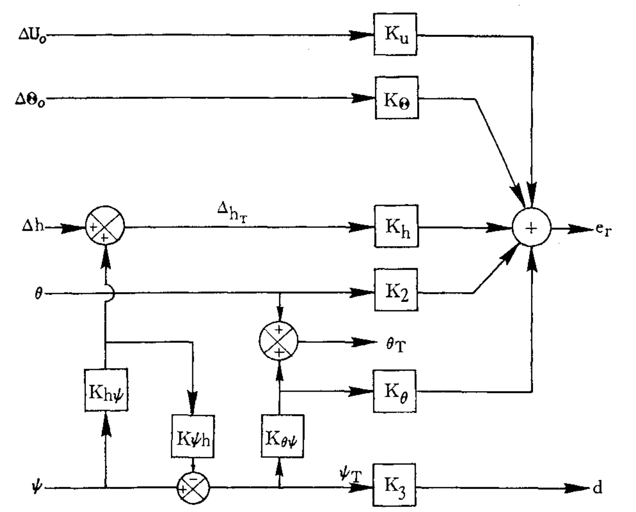

Fig. 3 Propagation of pilot errors into range error and deflection for an air-to-ground gun-firing pass. The measurable altitude and tracking errors are identified by the subscript *T*.

single-axis, mean-square tracking errors computed by applying analytical models of the (homogeneous) human pilot and turbulence to the longitudinal and lateral tracking dynamics. Instrumentation error can be readily included by appending a measurement error to the proper term. For example, let σ_{h_i} be the rms error between indicated altitude and true altitude, then the altitude term of Eq. (22) becomes $K_h{}^2(\sigma_h{}^2 + \sigma_{h_i}{}^2)$ since it is generally assumed that the instrument measurement errors are uncorrelated with pilot errors.

Application

The advantage of the weapon delivery model represented by Eqs. (22) and (23) is that it indicates simultaneously the relative importance of static and dynamic errors on impact accuracy. If the dive angle, airspeed, and altitude terms of Eq. (22) predominate, then it might be well to invest in a fire control system to reduce or automatically correct for these errors. On the other hand, if tracking error terms predominate, it would be more advantageous to alter the flight control system in order to improve the tracking dynamics.

For example, let us analytically determine the advantage of incorporating a head-up display (HUD) to indicate release altitude attainment instead of requiring the pilot to scan the altimeter. In addition, let us assume that the HUD, which need be nothing more than a "pickle command" light, incorporates a linear correction for dive angle errors. That is, the light is illuminated when the following altitude is reached:

$$h = h_n - (K_\Theta/K_h)\Delta\Theta_o \tag{25}$$

where h_n is the nominal, preselected weapon release altitude. The mission to be analyzed is strafing with nominal weapon release conditions of 520 knots, 500 ft, and a $-10°$ (stability axis) dive angle; hence, the sensitivities listed in Tables 1 and 2 apply.

The lateral tracking error is found to be minimum with the pure-gain pilot transfer function§

$$Y_{P_{A_F}} = 0.2666 \tag{26}$$

for pilot aileron control of the dynamics of a typical tactical fighter with its roll and yaw stability augmentation systems engaged. This is the pilot transfer function that would

§ The effect of the pilot's time delay was negligible at the low crossover frequency for lateral tracking by aileron control.

Table 3 Comparison of analytically-predicted, single-shot gun-firing accuracy with that measured from piloted simulation

Standard deviated	Scanned tracking with 0.5 dwell fraction		Full foveal tracking with θ_o-corrected HUD	
	Predicted	Measured	Predicted	Measured
Total longitudinal tracking error, $\sigma_{\theta T}$ deg[a]	0.47	0.56	0.42	0.31
Total lateral tracking error, $\sigma_{\psi T}$, deg[b]	0.73	0.73	0.65	0.65
Total release altitude error, σ_{hT}, ft	175	184	110	101
Horizontal range error of impact, σ_{er}, ft	137	155	118	98
Cross-tracking deflection of impact, σ_d, ft	35	45	30	35

[a] $\sigma_w = 4.224$ ft/sec for the white noise source producing longitudinal disturbances.
[b] $\sigma_w = 12$ ft/sec for the white noise source producing lateral distances.

represent full foveal tracking. If scanning of an altimeter were required, the effective pilot gain is reduced by the dwell fraction,[10] assumed here to be $\eta = 0.5$; hence, if scanning is required

$$Y_{PAS} = 0.1333 \tag{27}$$

With these two pilot models and a spanwise-varying vertical gust distribution to introduce lateral disturbances the rms lateral tracking error for full foveal tracking is

$$\sigma_{\psi F} = .0587\sigma_w, \text{ deg} \tag{28}$$

whereas, if the optical sight must be intermittently scanned, the lateral tracking error will be

$$\sigma_{\psi S} = .0662\sigma_w, \text{ deg} \tag{29}$$

where σ_w is the vertical gust intensity in ft/sec.

Combining the pilot-in-the-loop transfer functions for ψ, β, and ϕ according to Eq. (12) permits determination of the power spectrum density of the deflection, d. Integrating this power spectrum density for full foveal tracking gives:

$$\sigma_{dF} = 2.715\sigma_w, \text{ ft} \tag{30}$$

whereas, in the scanned case:

$$\sigma_{dS} = 3.170\sigma_w, \text{ ft} \tag{31}$$

Dividing Eqs. (30) and (31) by Eqs. (28) and (29), respectively, yields the effective deflection sensitivity to lateral tracking error for each case

$$K_{3F} = 46.25 \text{ ft/deg} \tag{32}$$

$$K_{3S} = 47.89 \text{ ft/deg} \tag{33}$$

Single-axis analysis of the longitudinal tracking situation using the techniques of Ref. 4 reveals that a pilot model of

$$Y_{PEF}(s) = 1.128(1 + 0.08749s)e^{-0.2s} \tag{34}$$

would permit a crossover frequency of 4.75 rad/sec for pilot elevator control of the dynamics of this typical tactical fighter with its pitch stability augmentation system engaged. However, if the longitudinal pilot model is to be homogeneous with the lateral pilot model, Eqs. (26) and (27) will have to apply to longitudinal tracking as well.¶

Using Eqs. (26) and (27) to also represent the pilot for foveal and scanned longitudinal tracking, and employing a lengthwise-varying vertical gust distribution to induce longitudinal disturbances, the rms longitudinal tracking error for full foveal and scanned tracking is

$$\sigma_{\theta F} = 0.0198\sigma_w, \text{ deg} \tag{35}$$

$$\sigma_{\theta S} = 0.0236\sigma_w, \text{ deg.} \tag{36}$$

where σ_w is the vertical gust intensity in fps.

¶ This is theoretically possible since crossover will still occur in a frequency region where the amplitude ratio has a -20 db/decade slope.

Combining the pilot-in-the-loop transfer functions for θ and $\Delta\alpha$ according to Eq. (14) permits determination of the power spectrum density of the dynamic portion of the range error, e_{rd}. Integrating the power spectrum density for the case of full foveal tracking gives:

$$\sigma_{e_{r_{dF}}} = 3.334\sigma_w, \text{ ft} \tag{37}$$

whereas, when scanning of the altimeter is required during tracking,

$$\sigma_{e_{r_{dS}}} = 3.880\sigma_w, \text{ ft} \tag{38}$$

Dividing Eqs. (37) and (38) by Eqs. (35) and (36), respectively, gives the effective range error sensitivity to longitudinal tracking error for each case:

$$K_{2F} = 168.4 \text{ ft/deg} \tag{39}$$

$$K_{2S} = 164.4 \text{ ft/deg} \tag{40}$$

The remaining undetermined parameters can be obtained from the empirical relations of Ref. 7 which were based on over 100 simulated gun-firing passes:

$$K_{\theta\psi} = 0.63 \tag{41}$$

$$K_{h\psi}\omega_\psi = -10.25 \text{ ft/deg-sec} \tag{42}$$

$$\eta_a\sigma_h = 69.2 \text{ ft} \tag{43}$$

$$1 - K_{h\psi}K_{\psi h} = .921 \tag{44}$$

$$\sigma_{U_o} = 6.90 \text{ fps} \tag{45}$$

$$\sigma_{\Theta_o} = 1.044 \text{ deg} \tag{46}$$

where ω_ψ = lateral tracking bandwidth
= 0.1055 rad/sec for foveal tracking, 0.0753 rad/sec for scanned tracking.

and η_a = altimeter dwell fraction
= 1.0 for HUD 0.5 for scanned altimeter.

In Table 3 predicted strafing accuracy results based on the foregoing analysis are compared with those measured in a piloted, hybrid-computer strafing simulation. Both the analytical and the simulator results show the same improvement trends with the addition of the HUD. In formulating the predictions it was assumed that the effect of the HUD was to eliminate the $K_\Theta{}^2\sigma_{\Theta_o}{}^2$ term from Eq. (22) and add the term $(K_\Theta/K_h)^2\sigma_{\Theta_o}{}^2$ to Eq. (24) due to the automatic correction of release altitude for dive angle errors. The turbulence intensities used in the simulation were chosen to provide as much tracking disturbance as possible without being objectionable to the experienced combat pilots. This was done to insure that the propagation of tracking errors into miss distance was properly modelled, since that was the main purpose of the simulation.

The actual vertical gust intensity that will be encountered at any altitude is itself a random variable. Using the turbulence data in Ref. 9, the mean value of $\sigma_w{}^2$ can be calculated as

$$\langle\sigma_w{}^2\rangle = E(\sigma_w{}^2) = 10.58P_1, \text{ ft}^2/\text{sec}^2 \tag{47}$$

Table 4 **Comparison of analytically-predicted, single-shot gun-firing accuracy with flight test data**

Standard deviation	Full foveal tracking Predicted[a]	Measured[b]
Total longitudinal tracking error, $\sigma_{\theta T}$, deg	.10	.17
Total lateral tracking error, $\sigma_{\psi T}$, deg	.14	.19
Total release altitude error, σ_{hT}, ft	71	67
Horizontal range error of impact, σ_{er}, ft	28	49
Cross-track deflection of impact, σ_d, ft	6.3	7.9

[a] $\sigma_w = 2.52$ ft/sec for both white noise sources.
[b] Calculated from the mean point of burst impact (corrected for wind and boresight error) for each of 10 passes.

where P_1 = probability of encountering turbulence at the weapon release altitude. The probability of encountering turbulence at 500 ft above ground level is 0.6 which establishes the appropriate mean for the rms vertical gust intensity as $\sigma_w = 2.52$ ft/sec from Eq. (47). Substituting this value into the preceding strafing model gives results comparable with actual flight test data as shown in Table 4. The important analytical outcome here is the determination of the relative contribution of each of the terms in Eq. (22) that yield the value of σ_{e_r} in Table 4; that is,

$$\sigma_{e_r}{}^2 = K_u{}^2\sigma_{U_o}{}^2 + K_\Theta{}^2\sigma_{\Theta_o}{}^2 + K_h{}^2\sigma_h{}^2 + K_2{}^2\sigma_\theta{}^2 + K_4{}^2\sigma_\psi{}^2 \quad (22)$$

$$(27.8)^2 = (3.4)^2 + (5.4)^2 + (7.2)^2 + (8.4)^2 + (24.6)^2 \quad (48)$$

From Eq. (48) it can be seen that tracking error is the greatest contributor to horizontal range error, and that most of the tracking error contribution comes from visual-motor coupling with the lateral tracking task. Hence, the best way to reduce both range error and deflection to improve the strafing accuracy of this aircraft is apparently to improve the lateral tracking dynamics with appropriate flight control system alterations. Two proposed techniques for accomplishing this are being analytically compared at the present time.

Conclusion

The preceding example illustrates a major advantage of the weapon delivery analysis approach presented here; that is, it gives some insight to the source of the major contributions to miss distance in each application. Consequently, redesign attention can be focused on problems whose solution will provide the greatest incremental improvement. It is not intended that the analytical approach described here supplant comprehensive piloted simulations in weapon system development, but it should provide a very useful analytical explanation of the results now obtained chiefly through flight test and extensive ground simulation. The system design deficiencies uncovered by piloted simulation studies can be more efficiently and effectively corrected if such an analytical model of the entire piloted system is available.

References

[1] McRuer, D. T. and Jex, H. R., "A Systems Analysis Theory of Manual Control Displays," *Third Annual NASA-University Conference on Manual Control*, NASA, SP-144, 1967, pp. 9–28.

[2] Clement, W. F., Jex, H. R., and D. Graham, "A Manual Control-Display Theory Applied to Instrument Landings of a Jet Transport," *Transactions on Man-Machine Systems*, Institute of Electrical and Electronics Engineers, Vol. 9, 1968, pp. 93–110.

[3] Jackson, V. R., "Control System Design for Maximum Effectiveness," AIAA Paper No. 70-342, St. Louis, Mo., 1970.

[4] McRuer, D. T. and Jex, H. R., "A Review of Quasi-Linear Pilot Models," *Transactions on Human Factors in Electronics*, Institute of Electrical and Electronics Engineers, Vol. 8, 1967, pp. 231–249.

[5] Blakelock, J. H., *Automatic Control of Aircraft and Missiles*, Wiley, New York, 1965, Chaps. 1 and 3.

[6] Preyss, A. E., Wiles, R. E., and Rogers, J. N., "An Analysis of the Precision Weapon Delivery Problem," Faculty Research Briefing, 1969, USAF Academy, Colorado Springs, Colo.

[7] Rankine, R. R., Jr., "The Effects of Aircraft Dynamics and Pilot Performance on Tactical Weapon Delivery Accuracy," Ph.D. dissertation, 1970, University of California, Los Angeles, Calif.

[8] Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, New York, 1965, Chap. 7.

[9] Chalk, C. R., Neal, T. P., Harris, T. M., Pritchard, F. E., and Woodcock, R. J., "Background Information and User Guide for MIL-F-8785 (ASG), 'Military Specification-Flying Qualities of Piloted Airplanes'," AFFDL-TR-69-72, Aug. 1969, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio, pp. 417–461.

[10] Levison, W. H. and Elkind, J. I., "Studies of Multivariable Manual Control Systems: Two-Axis Compensatory Systems with Separated Displays and Controls," CR-875, Oct. 1967, NASA.

[11] Allen, R. W., Clement, W. F., and Jex, H. R., "Research on Display Scanning, Sampling, and Reconstruction Using Separate Main and Secondary Tasks," TR 170-2, July 1969, Systems Technology, Inc., Hawthorne, Calif.

[12] Levison, W. H. and Elkind, J. I., "Studies of Multi-Variable Manual Control Systems: Two Axis Compensatory Systems with Compatible Integrated Display and Control," CR-554, Aug. 1966, NASA.